

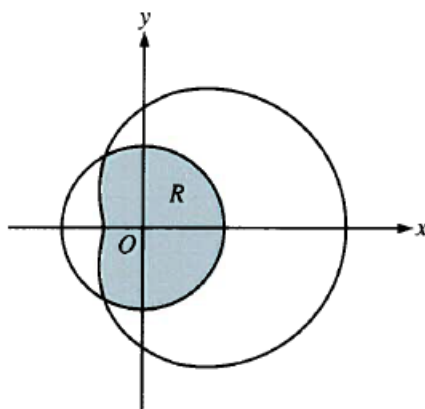
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AP Calculus BC – Polar Coordinate AP Problems

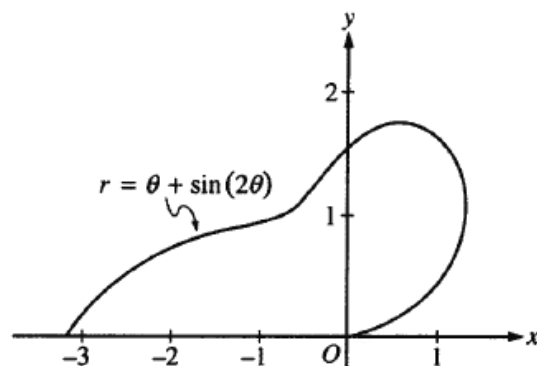
19. The area of the region inside the polar curve $r = 4 \sin \theta$ and outside the polar curve $r = 2$ is given by

- (A) $\frac{1}{2} \int_0^{\pi} (4 \sin \theta - 2)^2 d\theta$ (B) $\frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4 \sin \theta - 2)^2 d\theta$ (C) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4 \sin \theta - 2)^2 d\theta$
- (D) $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (16 \sin^2 \theta - 4) d\theta$ (E) $\frac{1}{2} \int_0^{\pi} (16 \sin^2 \theta - 4) d\theta$



The graphs of the polar curves $r = 2$ and $r = 3 + 2 \cos \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

- (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2 \cos \theta$, as shaded in the figure above. Find the area of R .



The curve above is drawn in the xy -plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

- Find the area bounded by the curve and the x -axis.
- Find the angle θ that corresponds to the point on the curve with x -coordinate -2 .
- For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r ? What does this fact say about the curve?
- Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.